interpretation of the experimental results, the assumption of a elastic-perfectly plastic material is valid.

The exterior surface of an open end cylinder subjected to internal pressure is in a condition of uniaxial stress since $\sigma_r$ and $\sigma_z$ are zero. Therefore, the condition of 100 percent overstrain may be defined as that at which the outside surface strain equals the strain associated with the yield stress of the material in uniaxial tension or

$$\epsilon_{tb} = \frac{\sigma_y}{E} \tag{3}$$

The pressure required to produce this condition ($P_0$) was experimentally determined for the specimens tested. It should be noted that, since most of the specimens were designed for greater than 100 percent overstrain, there was usually no contact between the specimen and container at the 100 percent overstrain condition. These values of $P_0$ were converted to pressure factor, and all values for the same diameter ratio were averaged and plotted in Fig. 7. These data may be represented by an empirical relationship:

$$P_0 = 1.08\,\sigma_y \log W \tag{4}$$

Weigle[1] gives the following equation for the 100 percent overstrain pressure based on the von Mises yield criterion and assuming $\sigma_z = 0$:

$$2\sqrt{(3)}\tan^{-1}|(\partial_0 - 1)^{\frac{1}{2}}| + \mathrm{lu}\,\partial_0 - \ln[1 + \sqrt{(3)}(\partial_0 - 1)^{\frac{1}{2}}]2 + \ln 3W_1^4 - \sqrt{(3)}\,\pi = 0 \tag{5}$$

where

$$\partial_0 = \frac{4K^2}{P_0^2}$$

and $K$ = yield stress in simple shear.

For ease of application, this equation may be approximated very accurately by the relationship

$$P_0 = 1.10\,\sigma_y \ln W_1 \tag{6}$$

It should be noted, however, that $W_1$ in Eq. (6) refers to the diameter ratio under pressure which is slightly less than the initial diameter ratio used in Eq. (4). Both equations, then, are in very close agreement and, for calculation purposes, the initial diameter ratio and Eq. (4) will be utilized.

The close agreement of Eq. (6) with the experimental data of Fig. 7 again verifies the assumption of an open end test condition.

*Partial Overstrain*

In deriving relationships for stresses and strains in a partially overstrained cylinder the following basic assumptions are made.

(*a*) The longitudinal stress throughout the tube is zero.

(*b*) The cylinder is considered infinitely long, since the center portion is far enough from the ends to be free of any end effect. The longitudinal strain, therefore, must be constant with respect to $r$.

(*c*) The pressure required to produce full plastic flow in the tube is given by Eq. (4), i.e., $P_0 = 1.08\,\sigma_y \log W$ where $W$ is the initial diameter ratio.

(*d*) If a cylinder is in the fully plastic condition (subject to an internal pressure $P_0$) and is then subjected to an external pressure, $P_{ex}$, in order to maintain the fully plastic condition and equilibrium, the internal pressure, $P_{in}$, must be increased by an amount equal to $P_{ex}$.

$$P_{in} = P_0 + P_{ex} \tag{7}$$

1. *Elastic region.* The elastic portion of a partially plastic cylinder is considered as a separate cylinder of internal radius $\rho$ (elastic-plastic interface radius) and external radius $b$. The radial stress and therefore the internal pressure at $\rho$ is equal to that required to produce elastic breakdown at $\rho$. Therefore, from Eq. (1):

$$\sigma_{r\rho} = -\,\sigma_y \frac{b^2 - \rho^2}{\sqrt{(3b^4 + \rho^4)}} \tag{8}$$

The stresses throughout the elastic region are then obtained from Eq. (8) and the Lamé equations

$$\sigma_{re} = \frac{\sigma_y \rho^2}{\sqrt{(3b^4 + \rho^4)}} \left(1 - \frac{b^2}{r^2}\right) \tag{9}$$

$$\sigma_{te} = \frac{\sigma_y \rho^2}{\sqrt{(3b^4 + \rho^4)}} \left(1 + \frac{b^2}{r^2}\right) \tag{10}$$

where $\rho \leq r \leq b$.

Strains in the elastic region are obtained from the above and the generalized Hookes' law.

$$\epsilon_{re} = \frac{\sigma_y \rho^2}{r^2 E \sqrt{(3b^4 + \rho^4)}} \left[(1-\mu)\, r^2 - (1+\mu)\, b^2\right] \tag{11}$$

$$\epsilon_{te} = \frac{\sigma_y \rho^2}{r^2 E \sqrt{(3b^4 + \rho^4)}} \left[(1-\mu)\, r^2 + (1+\mu)\, b^2\right] \tag{12}$$

$$\epsilon_z = -\,\frac{2\mu\, \sigma_y\, \rho^2}{E \sqrt{(3b^4 + \rho^4)}} \tag{13}$$

2. *Plastic region.* The plastic portion of the cylinder, i.e. where $a \leq r \leq \rho$, may be considered as a separate, fully plastic cylinder acted on by an internal